RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2015

FIRST YEAR [BATCH 2015-18]

PHYSICS [Hons]

Paper : I

Date : 14/12/2015

Time : 11 am – 3 pm

Full Marks : 100

[Use a separate Answer Book for each Group]

Group – A

Answer **any seven** questions taking at least **three** from each unit

Unit - I

1. a) Find the eigen values of the matrix (2)

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}^3$$

b) If λ be an eigen value of matrix A (non zero matrix). Show that λ^{-1} is an eigen value of A^{-1}. (1)

c) If a matrix is both Hermitian and unitary, show that all its eigen values are ±1. (2)

d) Show that the product of two symmetric matrices is symmetric only if they commute. (2)

e) If $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix}$, show that $(A+B)(A-B) \neq A^2 - B^2$. (3)

2. a) Solve the Differential Equation (4)

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

b) Solve the hypergeometric equation by series method (6)

$$x(1-x)\frac{d^2y}{dx^2} + \{\gamma - (\alpha+\beta+1)x\}\frac{dy}{dx} - \alpha\beta y = 0$$

3. a) What is the physical interpretation of divergence of a vector field? (2)

b) Show that $\vec{A}\cdot\frac{d\vec{A}}{dt} = A\frac{dA}{dt}$. (1)

c) Use the divergence theorem to evaluate $\oint_s \vec{A}\cdot d\vec{S}$ where $\vec{A} = (2x-z)\hat{i} + x^2y\hat{j} + xz^2\hat{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (3)

d) Using Green's theorem evaluate $\oint_c (x^2y\,dx + x^2dy)$ where c is the boundary describe counter clockwise of the triangle with vertices (0,0), (1,0), (1,1) then verify Green's theorem. (2+2)

4. a) Express the divergence operator in spherical polar coordinates starting from the expression of the same in Cartesian coordinates. (3)

b) State the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$ (2)

c) Express $\vec{\nabla}\times\vec{A}$ in cylindrical coordinate. (3)

d) Using Stokes theorem, prove that $\vec{\nabla}\times\vec{\nabla}\phi = 0$. (2)

5. a) Suppose u_1, u_2, u_3 are orthogonal curvilinear coordinates and x,y,z rectangular coordinates, then show that Jacobian $J\left(\frac{x,y,z}{u_1,u_2,u_3}\right)$ is equal to h_1,h_2,h_3 where h_1,h_2,h_3 are scale factors. (3)

b) Show whether the vectors {(1, –2, 3), (2, 3, 1), (–1, 3, 2)} form a basis or not of the vector space v^3. (3)

c) Construct an orthonormal vector set for R^3 out of the vectors $\Psi_1 = (1,1,1),\ \Psi_2 = (6,4,5),\ \Psi_3 = (3,6,9)$. (4)

Unit – II

6. a) A particle moves along a curve $x = 2t^2, y = 4t^2 - 1, z = 5t - 3$, where t denotes time. Find the components of acceleration at time t = 1 in the direction $(\hat{i} - 2\hat{j} + 2\hat{k})$. (3)

b) Find the (i) unit tangent vector $\hat{\mathrm{T}}$, (ii) acceleration vector $\vec{a}$ and (iii) curvature k to the space curve $\vec{r}(t) = t\hat{i} + \frac{t^2}{2}\hat{j} + t\hat{k}$, at any time t. (3)

c) State and prove work-energy theorem for a particle moving in a force field $\vec{F}$. If the force is partly conservative and partly non-conservative, what form does the theorem take, if the particle (4)

(i) moves from point P_1 to point P_2

(ii) moves around the closed path?

7. a) A particle of mass m moves along the x-axis under a resistive force $\vec{F} = -kv^2\hat{i}\ (k > 0)$. At time t = 0, its position is x = 0, and speed is v_0. Find

(i) the velocity at time t;

(ii) position at time t.

Can the particle ever come to rest? Explain. (4)

b) A particle is projected vertically upwards with an initial speed u_0. The restive force offered by the medium is kv per unit mass, where k > 0, and v is the instantaneous speed. Prove that the particle returns to the point of projection with speed u_1, given by $u_1 + u_0 = \frac{g}{k}\ln\left(\frac{g + ku_0}{g - ku_1}\right)$. (6)

8. a) Set up the general differential equation of motion of a body of variable mass $m(t)$ moving in a force field $\vec{F}$. Interpret the structure of the equation in physical terms. (4)

b) How is the above equation modified in the following cases:

(i) a rocket having a constant exhaust velocity $\vec{c}$?

(ii) a raindrop of instantaneous mass m falling vertically through a stationary cloud? (2)

c) A rocket of initial mass m_0 starts from rest in free space, with an exhaust velocity $\vec{c}$. For what value of the residual mass m is the momentum of the rocket maximum? What is the value of this maximum momentum? (4)

9. a) Find the center-of-mass of a uniform plate bounded by y = sin x and the x-axis, and lying in the first quadrant only. (2)

b) Prove that the total external torque on a system of particles is equal to the time rate of change of angular momentum of the system, provided that the internal forces between the particles are central forces. (2)

c) Show that when a conservative force acts on a system of particles, the total energy of the system is conserved. (2)

d) Three balls A, B and C of masses m, m and M respectively are kept on a frictionless table, along a straight line. Ball A moves with velocity V_A towards ball B and collides elastically. Ball B moves towards ball C and collides elastically. Show that if $m \geq M$ there will be two collisions, if $m < M$ there will be three collisions. (4)

10. a) A particle of mass 5g moves along the x-axis under the influence of two forces: (i) a force of attraction to the origin O which in dynes, is numerically equal to 40 times the instantaneous distance from 0, and (ii) a damping force proportional to the instantaneous speed such that when the speed is 10cm/s the damping force is 200 dynes. Assuming that the particle starts from rest at a distance 20 cm from 0, (6)

(i) Set up the differential equation and the conditions describing the motion.

(ii) Find the position of the particle at time t.

(iii) Find the total energy of the particle over a time period.

b) Show that the time-averaged power input and the time-averaged power dissipated in a forced oscillator are equal. (4)

Group - B

Answer **any three** questions

11. a) What is optical path? Deduce the one dimensional equation for ray path when the refractive index is a function of x only. [1+3]

b) An object is placed at a distance u from a refracting spherical surface and the image is formed at a distance v. Find the system matrix of the optical path. [4]

c) Find the magnification in the above problem if the refractive index in object side be n_1 and that in image side be n_2. [2]

12. a) Deduce system matrix for a thick lens of refractive index μ. [Thickness of lens is t, R_1, R_2 be the radii of curvature of two refracting surfaces] [5]

b) Use above matric to find the Lens maker formula for thin lens. [5]

13. a) Define chief ray and oblique centric pencil. Show them in a neat diagram. [3]

b) What is spherical aberration? How can it be reduced using two lenses separated by a distance? [1+3]

c) What is astigmatism? State any two methods for reduction of astigmatism. [1+2]

14. a) What do you mean by axial chromatic error in a lens? Show that the axial chromatic error for parallel incident rays is the product of the dispersive power and mean focal length for yellow light. [1+3]

b) A converging achromat of 40 cm focal length is to be constructed out of a thin crown and flint glass lens, the surfaces in contact having a common radius of curvature of 25 cm. Calculate the radius of curvature of the second surface of each lens, given that the values of the dispersive powers and refractive indices are 0·017 & 1·5 for crown glass, and 0·034 and 1·7 for flint glass. [4]

c) What kind of achromatism is required in photographic camera and eyepiece? [2]

15. a) What do you mean by Aperture stop and Exit pupil? [2]

b) Compare the action of Huygen's eyepiece and Ramsden's eyepiece in reference to aberration and measurement of objective image. [3]

c) Using matrix method determine the principal points of a Huygen's eyepiece. Show them in a neat diagram. [3+2]

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